Regular Maps

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February 18, 2023

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Introduction

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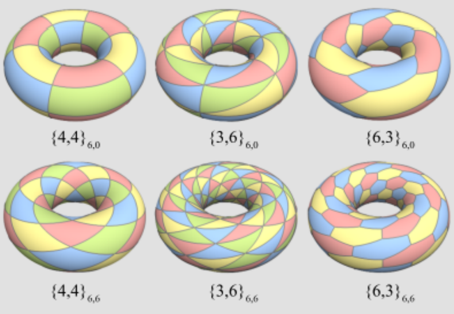
8 }0X«e9} h9q 9§ »} § Xe9jH0 _ 9 X}9q\ 0h@0LL0L _j 9 Hnhq9H§ 2 h9j_TneLJ «H\ §\9§ 0®0}´ T9H0 _ \nh0nhn}q_H §n 9 L_ c 9jL _§ 9«§nhn}q_ h X}n«q 9H§ }0X«e9}e´ nj ¼9X r0LX0J ®0}§0²J T9H0 §«qe0 su [0}0 }0X«e9}e´ h09j §\9§ §\0}0 _ 0²9H§e´ nj0 9«§nhn}q_ h h9qq_jX 9j´ ¼9X §n 9j´ n§\0}u

Introduction

8 }0X«e9} h9q 9§ »} § Xe9jH0 _ 9 X}9q\ 0h@0LL0L _j 9 Hnhq9H§ 2 h9j_TneLJ «H\ §\9§ 0®0}´ T9H0 _ \nh0nhn}q_H §n 9 L_ c 9jL _§ 9«§nhn}q_ h X}n«q 9H§ }0X«e9}e´ nj ¼9X r0LX0J ®0}§0²J T9H0 §«qe0 su [0}0 }0X«e9}e´ h09j §\9§ §\0}0 _ 0²9H§e´ nj0 9«§nhn}q_ h h9qq_jX 9j´ ¼9X §n 9j´ n§\0}u



Examples

d_cOe´ §\O »} § O²9 h qeO Tn«jL °O}O §\O qe9§nj_H ne_L °_H\ 9}O 9ee }OX«e9} h 9q nj §\O q\O}Ou

Examples

d_c0e´ §\0 »} § 0²9 h qe0 Tn«jL °0}0 §\0 qe9§nj_H ne_L °_H\ 9}0 9ee }0X«e9} h 9q nj §\0 q\0}0u

|0X«e9}_§´ _h qe_0 °0 H9j 9 nH_9§0 9 «j_x«0 9«§nhn}q_ h §n 0®0}´ ¼9X nT §\0 h 9qJ n h 9q 9}0 _j 9 °9´ §\0 G9´e0´ X}9q\ nT §\0_} n°j 9«§nhn}q_ h X}n«qu S«}§\0}hn}0 9j´ h 9q 9«§nhn}q_ h X}n«q _ 3 X0j0}9§0L @´ }0¼0H§_nj u

Examples

d_cOe´ §\0 »} § 0²9 h qe0 Tn«jL °0}0 §\0 qe9§nj_H ne_L °_H\ 9}0 9ee }0X«e9} h9q nj §\0 q\0}0u

|0X«e9}_§´ _h qe_0 °0 H9j 9 nH_9§0 9 «j_x«0 9«§nhn}q_ h §n 0®0}´ ¼9X nT §\0 h9qJ n h9q 9}0 _j 9 °9´ §\0 G9´e0´ X}9q\ nT §\0_} n°j 9«§nhn}q_ h X}n«qu S«}§\0}hn}0 9j´ h9q 9«§nhn}q_ h X}n«q _ 3 X0j0}9§0L @´ }0¼0H§_nj u

~n X_®0j 9 h9q °0 H9j Hnj §}«H§ _§ 9«§nhn}q_ h X}n«q 9jL X_®0j §\0 X}n«q °0 H9j Hnj §}«H§ §\0 9 nH_9§0L h9qu

Triangle groups

|0X«e9} h9q \9®0 9jn§\0} L0 H}_q§_nj _j §0}h nT §}_9jXe0 X}n«q u ¯ _H\ 9}0 X}n«q X0j0}9§0L @´ ¨ }0¼0H§_nj _j §\0 qe9j0 nT 9jXe0 $\left(\frac{}{l}; \frac{}{k}; \frac{}{m}\right)$ n °0 L0jn§0 «H\ 9 X}n«q @´

Triangle groups

|0X«e9} h9q \9®0 9jn§\0} L0 H}_q§_nj _j §0}h nT §}_9jXe0 X}n«q u ¯ _H\ 9}0 X}n«q X0j0}9§0L @´ ¨ }0¼0H§_nj _j §\0 qe9j0 nT 9jXe0 $(\frac{}{l}; \frac{}{k}; \frac{}{m})$ n °0 L0jn§0 «H\ 9 X}n«q @´

$$4\,(l;k;m) = hx;y;z \; j \; x^2 = y^2 = z^2 = (xy)^l = (yz)^k = (zx)^m = 1i$$

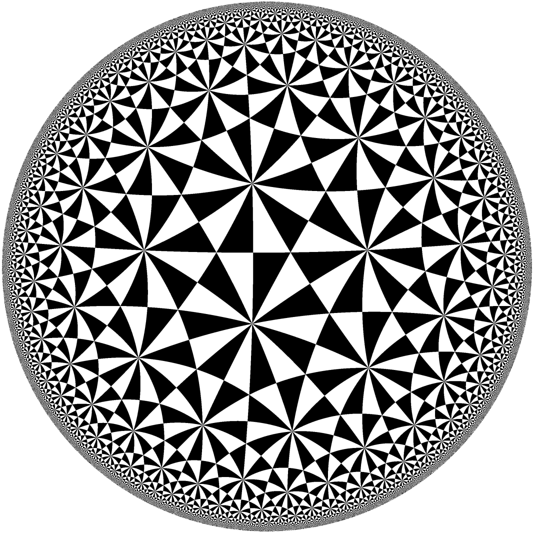
Triangle groups

|0X«e9} h9q \9®0 9jn§\0} L0 H}_q§_nj _j §0}h nT §}_9jXe0 X}n«q u ¯ _H\ 9}0 X}n«q X0j0}9§0L @´ ¨ }0¼0H§_nj _j §\0 qe9j0 nT 9jXe0 $(\frac{}{l}; \frac{}{k}; \frac{}{m})$ n °0 L0jn§0 «H\ 9 X}n«q @´

$$\Delta(l; k; m) = \langle x; y; z \mid x^2 = y^2 = z^2 = (xy)^l = (yz)^k = (zx)^m = 1 \rangle$$

¦ \0 0 X}n«q T9ee _j§n 3 H9§0Xn}_0 u ~q\0}_H9eJ N«He_L09j n} [´q0}@ne_Hu ¦ _ Hn}}0 qnjL §n °\0§\0} $\frac{1}{k} + \frac{1}{l} + \frac{1}{m}$ _ X}09§0} §\9j r~q\0}_H9es 0x«9e §n rNee_q§_Hs n} e0 §\9j o r[´q0}@ne_Hsu

Triangle groups



Triangle group quotients are maps

i n° §9c0 9 §}_9jXe0 X}n«q $4(2; m; k)$ _u0 9 }_X\§ §}_9jXe0 §_e_jX nT n h 0 q9H0u 8 }0X«e9} h9q _ _h qe´ 9 x«n§_0j§ nT «H\ 9 X}n«q @´ 9 §n} _nj T}00 jn}h9e «@X}n«q °_H\ q}0 0}®0 §\0 n}L0} m 9jL ku

Triangle group quotients are maps

i n° §9c0 9 §}_9jXe0 X}n«q 4 (2; *m*; *k*) _u0 9 }_X\§ §}_9jXe0 §_e_jX nT n h 0 q9H0u 8 }0X«e9} h 9q _ _h qe´ 9 x«n§_0j§ nT «H\ 9 X}n«q @´ 9 §n} _nj T}00 jn}h9e «@X}n«q °_H\ q}0 0}®0 §\0 n}L0} *m* 9jL *k*u

¦n 0²qe_H_§e´ Hnj §}«H§ §\0 0LX0 9jL ®0}§0² Hnjj0H§_nj °0 §9c0 §\0 }_X\§ Hn 0§ nT §\0 _h 9X0 nT *hxyi*J *hyzi* J *hzxi* _j §\0 x«n§_0j§ °_§\ _jH_L0jH0 X_®0j @´ jnj 0h q§´ _j§0} 0H§_nju

Triangle group quotients are maps

i n° §9c0 9 §}_9jXe0 X}n«q $4(2; m; k)$ _u0 9 }_X\§ §}_9jXe0 §_e_jX nT n h 0 q9H0u 8 }0X«e9} h 9q _ _h qe´ 9 x«n§_0j§ nT «H\ 9 X}n«q @´ 9 §n} _nj T}00 jn}h9e «@X}n«q °_H\ q}0 0}®0 §\0 n}L0} m 9jL ku

¦n 0²qe_H_§e´ Hnj §}«H§ §\0 0LX0 9jL ®0}§0² Hnj j0H§_nj °0 §9c0 §\0 }_X\§ Hn 0§ nT §\0 _h 9X0 nT $hxyi$J $hyzi$ J $hzxi$ _j §\0 x«n§_0j§ °_§\ _jH_L0jH0 X_®0j @´ jnj 0h q§´ _j§0} 0H§_nju

S}n h §_ q0} q0H§_®0 §\0 §\0n}´ nT }0X«e9} h 9q _ 0²9H§e´ 0x«_®9e0j§ §n §\0 §\0n}´ nT Ÿ h nn§\Ÿ x«n§_0j§ nT $f2; m; kg$ §}_9jXe0 X}n«q u [0}0 h nn§\ h 09j §\9§ §\0 x«n§_0j§ q}0 0}®0 §\0 n}L0} nT $yz; zx; xy$u

$$G = hx; y; zjx^2 = y^2 = z^2 = (yz)^k = (zx)^m = (xy)^2 = \quad = 1i$$

Reflexible and Chiral

d0§ M @0 9 h9q §\0j °0 L0jn§0 _§ 9«§nhn}q_ h X}n«q @´ $Aut(M)$ 9jL °0 L0jn§0 _§ n}_0j§9§_nj q}0 0}®_jX 9«§nhn}q_ h @´ $Aut^+(M)$. i n§0 §\9§ $Aut^+(M)$ \9 _jL0² 9§ hn § 2 _j $Aut(M)$.

Reflexible and Chiral

dO§ M @O 9 h9q §\Oj °O LOjn§O _§ 9«§nhn}q_ h X}n«q @´ $Aut(M)$ 9jL °O LOjn§O _§ n}_Oj§9§_nj q}O O}®_jX 9«§nhn}q_ h @´ $Aut^+(M)$u i n§O §\9§ $Aut^+(M)$ \9 _jLO² 9§ hn § 2 _j $Aut(M)$u

~nhO§_hO h9q 9Lh_§ 9j 9«§nhn}q_ h °_H\ _ 9 }O¼OH§_njJ _uO jnj n}_Oj§9§_nj q}O O}®_jX §\Oj $[Aut(M) : Aut^+(M)] = 2$ 9jL °O H9ee «H\ 9 h9q }O¼O²_@eO n§\O}°_ O °O H9ee _§ H_}9eu

A useful formula

¯0 jn° L0}_®0 9 j0H0 9}´ HnjL_§_nj Tn} 9 *fk; mg* h9q §n 0²_ §

A useful formula

¯0 jn° L0}_®0 9 j0H0 9}´ HnjL_§_nj Tn} 9 *fk; mg* h9q §n 0²_ §

$$jGj = 4E = 2mF = 2kV$$

A useful formula

¯0 jn° L0}_®0 9 j0H0 9}´ HnjL_§_nj Tn} 9 $fk; mg$ h9q §n 0²_ §

$$jGj = 4E = 2mF = 2kV$$

N«e0}ȴ Tn} h «e9 2 $2g = V \quad E + F$ X_®0 « §\0 0x«9§_njJ

$$2 \quad 2g = \frac{jGj}{2} \quad \frac{1}{k} + \frac{1}{m} \quad \frac{1}{2}$$

A useful formula

¯0 jn° L0}_®0 9 j0H0 9}´ HnjL_§_nj Tn} 9 $fk; mg$ h9q §n 0²_ §

$$jGj = 4E = 2mF = 2kV$$

N«e0}ꞁ Tn} h «e9 2 $2g = V$ $E + F$ X_®0 « §\0 0x«9§_njJ

$$2 \quad 2g = \frac{jGj}{2} \quad \frac{1}{k} + \frac{1}{m} \quad \frac{1}{2}$$

m} Tn} §\0 n}_0j§9§_nj q}0 0}®_jX H9 0

$$2 \quad 2g = jG^{+}j \quad \frac{1}{k} + \frac{1}{m} \quad \frac{1}{2}$$

A special family of maps

Hurwitz maps

¦n X0§ nh0 0²9hqe0 °0 H9j ennc 9§ nj0 _j§0}0 §_jX T9h_e´ nT h9q J §\0 n H9ee0L [«}°_§¶ h9q ıı ¦\0 0 Hnh0 T}nh §\0 }09e_ 9§_nj @´ [«}°_§¶ §\9§ §\0 n}L0} nT §\0 HnjTn}h9e n}_0j§9§_nj q}0 0}®_jX 9«§nhn}q_ h X}n«q nT 9j´ «}T9H0 nT X0j« g _ @n«jL0L 9@n®0 @´ $84(g - 1)$ıı

Hurwitz maps

¦n X0§ nh0 0²9hqe0 °0 H9j ennc 9§ nj0 _j§0}0 §_jX T9h_e´ nT h9q J§\0 n H9ee0L [«}°_§¶ h9q u ¦\0 0 Hnh0 T}nh §\0 }09e_ 9§_nj @´ [«}°_§¶ §\9§ §\0 n}L0} nT §\0 HnjTn}h9e n}_0j§9§_nj q}0 0}®_jX 9«§nhn}q_ h X}n«q nT 9j´ «}T9H0 nT X0j« g _ @n«jL0L 9@n®0 @´ $84(g - 1)$u

$$2 - 2g = |G^+|\left(\frac{1}{k} + \frac{1}{m} - \frac{1}{2}\right)$$

Hurwitz maps

¦n X0§ n h 0 0²9 h qe0 °0 H9j ennc 9§ nj0 _j§0}0 §_jX T9h_e´ nT h9q J §\0 n H9ee0L [«}°_§¶ h9q u ¦ \0 0 Hn h 0 T}n h §\0 }09e_ 9§_nj @´ [«}°_§¶ §\9§ §\0 n}L0} nT §\0 HnjTn}h9e n}_0j§9§_nj q}0 0}®_jX 9«§nhn}q_ h X}n«q nT 9j´ «}T9H0 nT X0j« g _ @n«jL0L 9@n®0 @´ $84(g - 1)$u

$$2 - 2g = |G^+|\left(\frac{1}{k} + \frac{1}{m} - \frac{1}{2}\right)$$

$$(2 - 2g)\left(\frac{1}{k} + \frac{1}{m} - \frac{1}{2}\right)^{-1} = |G^+|$$

Hurwitz maps

¦n XO§ nhO O²9hqeO °O H9j ennc 9§ njO _j§O}O §_jX T9h_e´ nT h9q J §\O n H9eeOL [«}°_§¶ h9q u ¦\O O HnhO T}nh §\O }O9e_ 9§_nj @´ [«}°_§¶ §\9§ §\O n}LO} nT §\O HnjTn}h9e n}_Oj§9§_nj q}O O}®_jX 9«§nhn}q_ h X}n«q nT 9j´ «}T9HO nT XOj« g _ @n«jLOL 9@n®O @´ $84(g \quad 1)$u

$$2 \quad 2g = jG^{+}j \quad \frac{1}{k} + \frac{1}{m} \quad \frac{1}{2}$$

$$(2 \quad 2g) \quad \frac{1}{k} + \frac{1}{m} \quad \frac{1}{2} \quad ^{1} = jG^{+}j$$

in° $\frac{1}{2}$ $\frac{1}{k}$ $\frac{1}{m}$ _ h_j_h_ OL °\Oj $m = 3$ 9jL $k = 7$ °\Oj _§ ®9e«O _ $\frac{1}{42}$ \OjHOJ

Hurwitz maps

¦n X0§ nh0 0²9hqe0 °0 H9j ennc 9§ nj0 _j§0}0 §_jX T9h_e´ nT h9q J §\0 n H9ee0L [«}°_§¶ h9q u ¦\0 0 Hnh0 T}nh §\0 }09e_ 9§_nj @´ [«}°_§¶ §\9§ §\0 n}L0} nT §\0 HnjTn}h9e n}_0j§9§_nj q}0 0}®_jX 9«§nhn}q_ h X}n«q nT 9j´ «}T9H0 nT X0j« g _ @n«jL0L 9@n®0 @´ $84(g \quad 1)$u

$$2 \quad 2g = jG^+j \quad \frac{1}{k} + \frac{1}{m} \quad \frac{1}{2}$$

$$(2 \quad 2g) \quad \frac{1}{k} + \frac{1}{m} \quad \frac{1}{2} \quad ^{1} = jG^+j$$

in° $\frac{1}{2} \quad \frac{1}{k} \quad \frac{1}{m}$ _ h_j_h_ 0L °\0j $m = 3$ 9jL $k = 7$ °\0j _§ ®9e«0 _ $\frac{1}{42}$ \0jH0J

$$84(g \quad 1) \quad jG^+j$$

Finding Hurwitz maps

[«}°_§¶ h9q 9}0 §\0}0Tn}0 §\0 h9q Tn} °_H\ §_ @n«jL _ 9§§9_j0LJ _u0 °\0j
$jG^{+}j = 84(g \quad 1)$u ?«§ Ln 9j´ 0®0j 0²_ §y

Finding Hurwitz maps

[«}°_§¶ h9q 9}0 §\0}0Tn}0 §\0 h9q Tn} °_H\ §_ @n«jL _ 9§§9_j0LJ _u0 °\0j $|G^+| = 84(g - 1)$u ?«§ Ln 9j´ 0®0j 0²_ §y

³0 R §\0 h9ee0 § X0j« [«}°_§¶ h9q _ §\0 9«§nhn}q_ h X}n«q nT §\0 be0_j x«9}§_HJ X_®0j @´ §\0 ¶0}n 0§ nT $x^3y + y^3z + z^3x$u ¦_ X}n«q \9 n}L0} $168 = 84(3 - 1)$ n _ X0j« ¨u

$$\langle R; S \mid R^3 = (RS)^2 = S^7 = (RS^{-2})^4 = 1 \rangle$$

Finding more Huwitz maps

Finding more Huwitz maps

dO§ G @O 9 [«}°_§¶ X}n«q nT XOj« g °_H\ Hn}}O qnjL §n 9 jn}h9e «@X}n«q K nT _jLO² $84(g \quad 1)$ _j $4(2;3;7)$u ¦\Oj §9cO §\O «@X}n«q dQ$[K;K]$ K^m §_ _ §\O «@X}n«q XOjO}9§OL @´ Hnh h«§9§n} 9jL m^{th} qn°O} _j Ku ¦_ «@X}n«q _ HeO9}e´ H\9}9H§O}_ §_H r_j®9}_9j§ «jLO} 9«§nhn}q_ hs 9jL §\O}OTn}O _ jn}h9e _j $4(2;3;7)$u

Finding more Huwitz maps

dO§ G @O 9 [«}°_§¶ X}n«q nT XOj« g °_H\ Hn}}O qnjL §n 9 jn}h9e «@X}n«q K nT _jLO² $84(g-1)$ _j $\Delta(2;3;7)$u ¦\Oj §9cO §\O «@X}n«q dQ$[K;K]$ K^m §_ _ §\O «@X}n«q XOjO}9§OL @´ Hnh h«§9§n} 9jL m^{th} qn°O} _j Ku ¦_ «@X}n«q _ HeO9}e´ H\9}9H§O}_ §_H r_j®9}_9j§ «jLO} 9«§nhn}q_ hs 9jL §\O}OTn}O _ jn}h9e _j $\Delta(2;3;7)$u

~_jHO K h« § @O 9 «}T9HO X}n«q _§ \9 q}O 0j§9§_nj _j §O}h nT $2g$ XOjO}9§n} $A_i; B_i$ Tn} $i \in \{1; \ldots ; g\}$ °_§\ njO }Oe9§_nj $\prod_{i}^{g}[A_i; B_i]$u w«n§Oj§_jX @´ L \9 §\O O¸OH§ nT h9c_jX §\O O XOjO}9§n} Hnh h«§O 9jL \9®O n}LO} m n

Finding more Huwitz maps

dO§ G @O 9 [«}°_§¶ X}n«q nT X0j« g °_H\ Hn}}0 qnjL §n 9 jn}h9e «@X}n«q K nT _jLO² $84(g \quad 1)$ _j $4(2;3;7)$u ¦\0j §9c0 §\0 «@X}n«q dQ[$K;K$] K^m §_ _ §\0 «@X}n«q X0j0}9§0L @´ Hnh h«§9§n} 9jL m^{th} qn°0} _j Ku ¦_ «@X}n«q _ He09}e´ H\9}9H§0}_ §_H r_j®9}_9j§ «jL0} 9«§nhn}q_ hs 9jL §\0}0Tn}0 _ jn}h9e _j $4(2;3;7)$u

~_jH0 K h« § @0 9 «}T9H0 X}n«q _§ \9 q}0 0j§9§_nj _j §0}h nT $2g$ X0j0}9§n} $A_i;B_i$ Tn} $i \; 2 \; f1;:::;gg$ °_§\ nj0 }0e9§_nj $\prod_{i \quad g}[A_i;B_i]$u w«n§0j§_jX @´ L \9 §\0 0¸0H§ nT h9c_jX §\0 0 X0j0}9§n} Hnh h«§0 9jL \9®0 n}L0} m n

$$K/L = \mathbb{Z}/m\mathbb{Z} \quad \mathbb{Z}/m\mathbb{Z} \qquad\qquad \mathbb{Z}/m\mathbb{Z}$$

°\0}0 §\0}0 9}0 $2g$ T9H§n} nT $\mathbb{Z}/m\mathbb{Z}$u ~n $[K:L] = m^{2g}$u

Finding more Huwitz maps

dO§ G @O 9 [«}°_§¶ X}n«q nT XOj« g °_H\ Hn}}O qnjL §n 9 jn}h9e «@X}n«q K nT _jLO² $84(g \quad 1)$ _j $4(2;3;7)$u ¦\Oj §9cO §\O «@X}n«q dQ$[K;K]$ K^m §_ _ §\O «@X}n«q XOjO}9§OL @´ Hnh h«§9§n} 9jL m^{th} qn°O} _j Ku ¦_ «@X}n«q _ HeO9}e´ H\9}9H§O}_ §_H r_j®9}_9j§ «jLO} 9«§nhn}q_ hs 9jL §\O}OTn}O _ jn}h9e _j $4(2;3;7)$u

~_jHO K h« § @O 9 «}T9HO X}n«q _§ \9 q}O 0j§9§_nj _j §O}h nT $2g$ XOjO}9§n} $A_i; B_i$ Tn} $i \ 2 \ f1; : : : ; gg$ °_§\ njO }Oe9§_nj $\prod_i^g [A_i; B_i]$u w«n§Oj§_jX @´ L \9 §\O O¸OH§ nT h9c_jX §\O O XOjO}9§n} Hnh h«§O 9jL \9®O n}LO} m n

$$K/L = Z/mZ \quad Z/mZ \qquad Z/mZ$$

° \O}O §\O}O 9}O $2g$ T9H§n} nT Z/mZu ~n $[K : L] = m^{2g}$u g O9j_jX §\9§ 9e n $[4(2;3;7) : L] = 84(g \quad 1) \quad m^{2g}$ \OjHO

$$j4(2;3;7)/Lj = 84(m^{2g}(g \quad 1) + 1)$$

The same trick for soluble maps

[illegible]

The same trick for soluble maps

¦\0 9@n®0 §OH\j_x«0 H9eeOL §\0 ļg 9H@09§\ ¦}_Hcļ : _ «}q}_ _jXe´ « OT«eJ _§ H9j 9e n @0 « OL §n \n° §\9§ §\0}0 9}0 Tn} 0²9hqe0 _j»j_§0e´ h9j´ ne«@eO h9q u

dO§ G @0 9 ne«@eO }OX«e9} h9q nT X0j« g °_H\ Hn}}0 qnjL §n 9 jn}h9e «@X}n«q K nT _jL0² n _j $T = 4(2; m; k)$u ¦\0j 9X9_j §9c0 §\0 «@X}n«q $L = [K; K]$ K^du 8X9_j °0 \9®0 §\9§ K/L _ 9@0e_9j r9jL §\0}0Tn}0 ne«@eOs 9jL T/L _ 9 }OX«e9} h9qu

The same trick for soluble maps

¦ \0 9@n®0 §OH\ j_x«0 H9eeOL §\0 ˛g 9H@09§\ ¦}_Hc˛ : _ «}q}_ _j Xe´ « 0T«eJ _§ H9j 9e n @0 « 0L §n \n° §\9§ §\0}0 9}0 Tn} 0²9 h qe0 _j »j_§0e´ h 9j ´ ne«@e0 h 9q u

d0§ G @0 9 ne«@e0 }0X«e9} h 9q nT X0j« g °_H\ Hn}}0 qnjL §n 9 jn}h 9e «@X}n«q K nT _jL0² n _j $T = 4(2; m; k)$u ¦ \0j 9X9_j §9c0 §\0 «@X}n«q $L = [K; K]$ K^du 8X9_j °0 \9®0 §\9§ K/L _ 9@0e_9j r9jL §\0}0Tn}0 ne«@e0s 9jL T/L _ 9 }0X«e9} h 9qu

¯0 °9j§ §n \n° §\9§ T/L _ ne«@e0u ?«§ §_ @0Hn h 0 He09} njH0 °0 9qqe´ §\0 2^{nd} X}n«q _ n h n}q_ h §\0n}0 h °_H\ §0ee « §\9§

The same trick for soluble maps

¦\0 9@n®0 §OH\j_x«0 H9eeOL §\0 ˛g9H@09§\ ¦}_Hc˛ : _ «}q}_ _jXe´ « OT«eJ _§ H9j 9e n @0 « OL §n \n° §\9§ §\0}0 9}0 Tn} 0²9hqe0 _j»j_§0e´ h9j´ ne«@e0 h9q u

d0§ G @0 9 ne«@e0 }0X«e9} h9q nT X0j« g °_H\ Hn}}0 qnjL §n 9 jn}h9e «@X}n«q K nT _jL0² n _j $T = 4(2; m; k)$u ¦\0j 9X9_j §9c0 §\0 «@X}n«q $L = [K; K]$ K^du 8X9_j °0 \9®0 §\9§ K/L _ 9@0e_9j r9jL §\0}0Tn}0 ne«@e0s 9jL T/L _ 9 }0X«e9} h9qu

¯0 °9j§ §n \n° §\9§ T/L _ ne«@e0u ?«§ §_ @0Hnh0 He09} njH0 °0 9qqe´ §\0 2^{nd} X}n«q _ nhn}q_ h §\0n}0h °_H\ §0ee « §\9§

$$(T/L)/(K/L) = T/K = G$$

The same trick for soluble maps

[illegible]

[illegible] G [illegible] g [illegible] K [illegible] n [illegible] $T = 4(2; m; k)$ [illegible] $L = [K; K]$ [illegible] K^d [illegible] K/L [illegible] T/L [illegible]

[illegible] T/L [illegible] 2^{nd} [illegible]

$$(T/L)/(K/L) = T/K = G$$

[illegible] G [illegible] K/L [illegible] T/L [illegible] $4(2; m; k)$ [illegible] d [illegible]

Density of Regular Maps

¦ \0 L0j _§´ nT 9 0§ nT qn _§_®0 _j§0X0} X N _ L0»j0L 9 Tneen° J

d0§ $X_n = X \cap \{1; 2; 3; \ldots; n\}$ §\0j §\0 L0j _§´ (X) nT X _ §\0 e_h_§

$$\delta(X) = \lim_{n \to \infty} \frac{|X|}{n}$$

_T §_ e_h_§ Ln0 jn§ 0²_ § °0 h9´ }0qe9H0 _§ °_§\ 0_§\0} e_h «q n} e_h_jTu

Density of Regular Maps

Conjecture (Thomas Tucker)

d0§ $X_{m;k}$ N @0 §\0 0§ nT X0j0}9 nT «}T9H0 Tn} ° _H\ 9L h_§ 9 }0X«e9} h9q nT §´q0 $f m; k g$ꭒ ¦ \0jJ

$$(X_{m;k}) = 0:$$

m} @´ 9@« 0 nT jn§9§_nj $(m; k)$ꭒ

m} 0x«_®9e0j§e´J §\0 L0j _§´ nT §\0 0§ nT _jL0²0 nT §n} _nj T}00 x«n§_0j§ nT §}_9jXe0 X}n«q _ ¶0}nꭒ

Progress so far

?´ 9 }0 «e§ nT ?0}§}9h rok¢£s °0 H9j q}n®0 §\9§ Tn} mJ kJ 2 }0e9§_®0e´ q}_h0 §\0j $(m; k) = 0$u

Theorem (Bertram's Theorem)

d0§ B @0 §\0 0§ nT j«h@0} b «H\ §\9§ 9j´ X}n«q G nT n}L0} b \9 9 jn}h9e H´He_H ´en° p «@X}n«q¦ \0j $(B) = 1$u

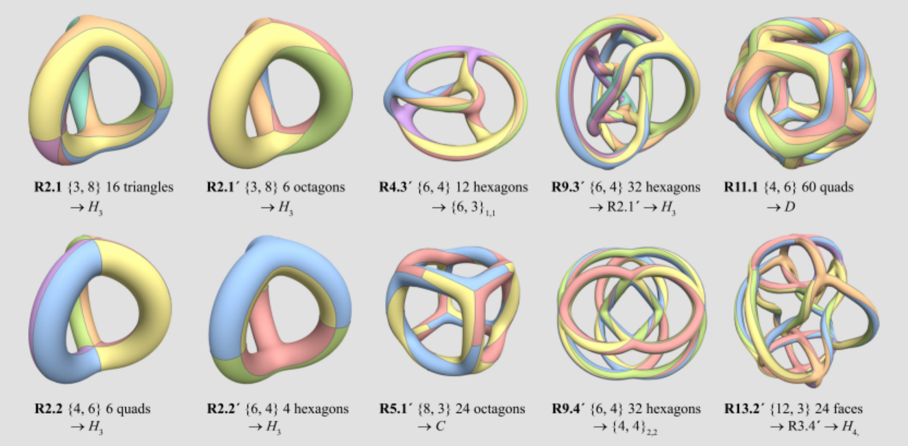
ª _jX nh0 }09 nj9@e´ 0e0h0j§9}´ X}n«q §\0n}´ °0 H9j \n° §\9§ jn }0X«e9} h9q °_§\ $m; k$ 9jL 2 }0e9§_®0e´ q}_h0 _ 9 ?0}§}9h X}n«q 9jL §\0}0Tn}0 §\0 L0j _§´ nT §\0_} n}L0} 9jL §\0}0Tn}0 @´ §\0 N«e0}]pn_jH9}0 Tn}h«e9 §\0 L0j _§´ nT §\0}0 X0j0}9 _ ¶0}nu

Density of soluble regular maps

8jn§\0} L0j _§´ }0e9§0L Hnja0H§«}0 r§\n«X\ §_ §_h0 °0 }0T0} §n L0j _§´ nT H0}§9_j §´q0 nT h9q °_§_j §\0 0§ nT }0X«e9} h9q 9 nqqn 0L §n L0j _§´ nT X0j« _j §\0 j9§«}9e s _ 9@n«§ §\0 L0j _§´ nT ne«@e0 }0X«e9} h9q J _§ \9 @00j H\0Hc0L Hnhq«§9§_nj9ee´ @´ g9} §nj GnjL0} §\9§ 9@n«§ 95 t nT 9ee n}_0j§9@e´]}0X«e9} h9q nT X0j« @0en° ¨·o 9}0 ne«@e0u

¦\0 }09 nj Tn} §_ _ jn§ 9§ 9ee n@®_n« 9jL §\n«X\ _§ _ Hnja0H§«}0L §\9§ §_ 9@«jL9jH0 nT ne«@e0 h9q Hnj§_j«0 Tn} _X\0} X0j« §\0}0 _ jn§ h«H\ §\0n}0§_H9e 0®_L0jH0 §n «qqn}§ §_ ´0§u ª _jX §\0 g9H@09§\ §}_Hc °0 \n°0L §\9§ _T §\0}0 0²_ § nj0 ne«@e0 h9q nT §´q0 $fm;kg$ §\0j §\0}0 9}0 _j»j_§0e´ h9j´u ^§ _ 9 h9ee q9}§ nT h´ p\K q}na0H§ §n q}n®0 §\9§ nj0 ne«@e0 h9q 0²_ § Tn} 9j´ \´q0}@ne_H h9q nT §´q0 $fm;kg$ °0 \9®0 9 h0§\nL §n Hnj §}«H§ ne«@e0 x«n§_0j§ §\9§ °0 @0e_0®0 °_ee 9e°9´ °n}c @«§ §\0 q}nnT _ jn§ ´0§ Hnhqe0§0u

Thank you!


R2.1 {3, 8} 16 triangles $\to H_3$
R2.1′ {3, 8} 6 octagons $\to H_3$
R4.3′ {6, 4} 12 hexagons $\to \{6, 3\}_{1,1}$
R9.3′ {6, 4} 32 hexagons $\to$ R2.1′ $\to H_3$
R11.1 {4, 6} 60 quads $\to D$
R2.2 {4, 6} 6 quads $\to H_3$
R2.2′ {6, 4} 4 hexagons $\to H_3$
R5.1′ {8, 3} 24 octagons $\to C$
R9.4′ {6, 4} 32 hexagons $\to \{4, 4\}_{2,2}$
R13.2′ {12, 3} 24 faces $\to$ R3.4′ $\to H_4$